A Hybrid Differential Evolution Method for the Design of IIR Digital Filter

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Abstract— **This paper establishes methodology for the robust and stable design of infinite impulse response (IIR) digital filters using hybrid differential evolution method. Differential Evolution (DE) is undertaken as a global search technique and exploratory search is exploited as a local search technique. DE is a population based stochastic real parameter optimization technique relating to evolutionary computation, whose simple yet powerful and straight forward features make it very attractive for numerical optimization. Exploratory search aims to fine tune the solution locally in promising search area. This proposed DE method augments the capability to explore and exploit the search space locally as well globally to achieve the optimal filter design parameters by applying the opposition learning strategy and random migration. A multivariable optimization is employed as the design criterion to obtain the optimal stable IIR filter that minimizes the magnitude approximation error and ripple magnitude. DE method is implemented to design low-pass, high-pass, band-pass, and band-stop digital IIR filters. The achieved design of IIR digital filters by applying DE method authenticates that its results are comparable to other algorithms and can be effectively applied for higher filter design.**

Index Terms—**Digital IIR filters, Differential Evolution, Exploratory search algorithm, Multi-parameter, optimization, Opposition based learning, Random migration.**

I. INTRODUCTION

The need for numerical optimization algorithms arises from almost every field of engineering, science, and business. This is because; an analytical optimal solution is difficult to obtain even for relatively simple application problems. A numerical algorithm is expected to perform the task of global optimization of a multimodal and non linear objective functions. However, an objective function possesses numerous local optima, which could trap the numerical algorithms. The possibility of failing to locate the desired global solution increases with the Increase in the problem dimension. The main challenge in designing infinite impulse response (IIR) digital filters is its nonlinearity and stability aspects in comparison to finite impulse response (FIR) digital filter design. The digital filters can be implemented through either hardware or software. These are capable of processing both real-time and on-line signals. Image processing, speech synthe-sis, secure communication, radar processing and biomedical etc are some of the areas where digital filters are useful.

The design of digital IIR filters is achieved by either transformation or optimization technique. In the transformation technique, analog IIR filter is designed and then it is converted to digital IIR filter. By implementing transformation techniques [10], Butterworth, Chebyshev, and Elliptic functions, have been designed. Optimization methods have been applied whereby magnitude error, and ripple magnitudes of pass-band and stop-band are undertaken to measure performance of digital IIR filter design. Jiang *et.al* [30] has discussed the design of IIR digital filter by comprising stability constraint and has applied an iterative second-order cone programming method. Lightener *et.al*.[3] has discussed the simultaneous design in magnitude and group delay. A stability constraint with a prescribed pole radius has been derived from the argument principle of complex analysis for designing IIR digital filter. The semi-definite programming relaxation technique [31] has been implemented for design of IIR filter in a convex form.

The design procedure is sequential and finds a feasible solution within a set of relaxed constraints. Normally conventional gradient-based design may easily get stuck in the local minima of error surface of IIR filters, due to its non-linear and multimodal nature. To overcome the shortcomings of gradient methods, researchers have applied nature based optimization algorithms such as genetic algorithms [4,6,8,9,11,15,16], particle swarm optimization (PSO) [17], seeker- optimization- algorithm -based evolutionary method [20], simulated annealing (SA) [12], Tabu search [19], ant colony optimization [18], immune algorithm [22] etc for the design of digital filters.

Natural selection and genetics have been the basis for evolutionary algorithms and these are used as promising optimization techniques. Genetic algorithm (GA) also falls in the category of evolutionary algorithms. GA is capable of searching multidimensional and multimodal decision spaces. These are also useful to optimize complex and discontinuous functions [8]. The digital IIR filter can be structured as cascade, parallel, or lattice. Tang *et al.*[8] have applied genetic algorithms for designing the digital IIR filters. Genetic algorithms sometimes encounter very slow convergence. For

high dimensional optimization problems GA may trap in the local optima of objective function due to the existence of numerous local optima [7]. Tsai *et al.* [21] have put on the hybrid Taguchi genetic algorithm for design of optimal IIR digital filters. In hybrid Taguchi genetic algorithm, Taugchi has applied the combination of the traditional genetic algorithms .Consequently it becomes necessary to explore an algorithm that may provide global solution for the design of the optimal digital IIR filters.

Tsai *et al.* [22] have suggested the integrating of the immune algorithm and the Taguchi method and named it as Taguchi-immune algorithm (TIA), for the design of digital IIR filter. Yu *et.al.* [24] have proposed cooperative co-evolutionary GA. During design of digital IIR filter, the lowest filter order has been found by considering the magnitude and the phase response. Here the digital IIR filter structure and its coefficients have been coded separately. The simulated annealing has been applied for the coefficient species so that diversity is maintained, for arriving at global minima [12], and it may require too many function evaluations. The seeker-optimization-algorithm is good at local convergence and it can be implemented for global minima [20]. It might often require too many objective function evaluations. The literature survey reveals that, there are various methods with which the optimization problems under different conditions can be addressed Optimization methods are classified based on the nature of objective function and type of search space. The evaluation of optimum solutions could be computationally expensive in IIR filter design problems due to time consuming computer simulation. Hence, a method is required, which is able to provide optimum solution within a given time budget.

Kennedy and Ebehart [5] developed a global search technique and named it as particle swarm optimization (PSO). It is based on simple concept, easy to implement, computationally fast and has robust search ability. In PSO the social evolvement knowledge is simulated by searching the optimum through population evolution which may include fittest solutions. Despite many advantages PSO has some shortcomings because the convergence behavior of PSO depends upon its parameters. By chance PSO parameters have been chosen incorrectly, this may result in divergent particle trajectories which results trapping into local minima [27]. Further high-dimensional optimization problems, may suffer from the premature convergence problem. Till date, many efforts have been made by the researchers to improve the performance of PSO, either through mathematical analysis or by improving the convergence property [14, 32].

In the past, to tackle numerically complex computational problems, population based algorithms, like evolutionary algorithms (EAs), particle swarm optimization, and differential evolution, have been developed by the researchers. However, the performance of these algorithms may vary extensively from one problem to another and there is no single best algorithm for all problems. Practically, there is an inherent risk associated while choosing an algorithm for the given problem, as in prior no researcher knows which algorithm will give optimal result for the problem. Further finding the optimum solution may not be practical or even possible for many real-world applications. The best solution obtained by a given iteration is often considered to be very important. In other words, an algorithm that performs extremely well on some problems but very poorly on other problems. So there is a more desirable that works well on a large variety of problems.

These days' evolutionary algorithms comprising of GA, EP, ES, (DE) and so on are mostly used for solving complex real life computational problems. Differential Evolution differs from conventional GA in its use of perturbing vectors, which are the difference between two randomly chosen parameter vectors, a concept borrowed from the operators of Nelder and Mead's simplex optimization technique. DE algorithm was first introduced by Storn and Price in 1995 and was successfully applied in the optimization of some well-known nonlinear, non-differentiable, and non-convex functions. Das and Suganthan [33] provided an overview of the engineering applications that have benefited from the powerful nature of DE. Differential evolution algorithm has a number of significant advantages. DE has the ability to find the true global minimum regardless of the initial parameter values. It is fast and simple with regard to application. It requires few control parameters. It has parallel processing nature and fast convergence. DE is capable of providing multiple solutions in a single run. The method is effective on integer, discrete and mixed parameter optimization. DE has ability to find the optimal solution for a nonlinear constrained optimization problem with penalty functions. Although DE has many advantages explained above, this has a number of disadvantages. In DE, there exist many trial vectors generation strategies out of which a few may be suitable for solving a particular problem. Ten mutation strategies are undertaken to add a vector differential to population vector of individuals. Moreover, three crucial control parameters involved in differential evolution algorithm, i.e., population size, scaling factor, and crossover rate, may significantly influence the optimization performance [26]. Hybrid algorithms have the scope to overcome the shortcomings of the evolutionary techniques.

The objective of this paper is to propose a hybrid DE method for the design of IIR digital filters that randomly explores the search space globally as well locally. The opposition employees for population initialization and for generation jumping The migration has been applied to maintain the diversity and search space exploration. Locally the solutions are fine tuned with exploratory search. The scaling factor is varied over generations by dynamic systems evidencing chaotic behavior. The values of the filter coefficients are optimized with the differential evolution to achieve magnitude error and ripple magnitude of pass-band and stop-band as objective functions for optimization problem. Here constraints are taken care of by applying exterior penalty method.

The paper is organized in four sections. Section 2 describes the IIR filter design problem statement. The

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underlying mechanism and details regarding the differential evolutionary algorithm for designing the optimal digital IIR filters is described in Section 3. The performance of the proposed differential evaluation method has been evaluated and achieved results are compared with the design results by Tang *et al.* [8], Tsai *et al.* [21] and Tsai and Horng [22] for the low pass, high pass, band pass, and band stop filters in section 4. Finally, in Section 5 the conclusions and discussions are outlined.

II. IIR FILTER DESIGN PROBLEM

In the digital filter design, a set of filter coefficients is determined, which should meet the performance specifications of pass-band width and its corresponding gain, stop-band width and its attenuation, band edge frequencies, and tolerable peak ripple in the pass-band and stop-band. The realization of IIR digital filter is stated by the recursive equation [34]:

$$y(n) = \sum_{k=0}^{N} x_k u(n-k) - \sum_{j=1}^{M} x_{N+j} y(n-j) \tag{1}$$

$$H(z) = \frac{\sum_{k=0}^{N} x_k z^{-k}}{1 + \sum_{j=1}^{M} x_{N+j} z^{-j}} \tag{2}$$

The design of digital filter design problem involves evaluation of a set of filter coefficients, x_k and x_{N+j} which meet the performance indices. Several first-order and second-order sections are cascaded together [7, 8] for realizing IIR filters. The cascaded transfer function of IIR filter is denoted by $H(\omega, x)$, involving the filter coefficients like, poles and zeros. For all types of filter, the structure of cascading type digital IIR filter is stated as [4].

$$H(\omega, x) = x_1 \left(\prod_{i=1}^{N} \frac{1 + x_{2i} e^{-j\omega}}{1 + x_{2i+1} e^{-j\omega}} \right) \times \left(\prod_{k=1}^{M} \frac{1 + x_l e^{-j\omega} + x_{i+1} e^{-2j\omega}}{1 + x_{l+3} e^{-j\omega} + x_{l+4} e^{-2j\omega}} \right) \tag{3}$$

where:

$l = 2N + 4(k-1) + 2$ and vector $X = [x_1 \ x_2 \ ... x_S]^T$ denotes the filter coefficients of dimension S×1 with $S = 2N + 4M + 1$. In the IIR filter, the coefficients are optimized such that the approximation error function for magnitude is to be minimized. The magnitude response is specified at K equally spaced discrete frequency points in pass-band and stop-band. The absolute error is denoted by $e(x)$ and is stated below:

$$e(x) = \sum_{i=0}^{K} \left| H_d(\omega_i) - |H(\omega_i, x)| \right| \tag{4}$$

Desired magnitude response, $H_d(\omega_i)$ of IIR filter is given as:

$$H_d(\omega_i) = \begin{cases} 1, & ; for \ \omega_i \in passband \\ 0, & ; for \ \omega_i \in stopband \end{cases} \tag{5}$$

The ripple magnitudes of pass-band and stop-band are given by $\delta_1(x)$ and $\delta_2(x)$, respectively [2]. Ripple magnitudes for pass-band is given as:

$$\delta_1(x) = \max_{\omega_i} \left\{ |H(\omega_i, x)| \right\} - \min_{\omega_i} \left\{ |H(\omega_i, x)| \right\} ; \omega_i \in passband \tag{6}$$

Ripple magnitudes for stop-band is stated as

$$\delta_2(x) = \max_{\omega_i} \left\{ |H(\omega_i, x)| \right\}; \omega_i \in stopband \tag{7}$$

Stability constraints are included in the design of casual recursive filters, which are obtained by using the Jury method [1]. The multivariable constrained optimization problem is stated as below:

Minimize $f(x) = e(x)$ (8a)

Subject to the stability constraints:-

$1 + x_{2i+1} \geq 0 \ (i = 1, 2, ..., N)$ (8b)

$1 - x_{2i+1} \geq 0 \ (i = 1, 2, ..., N)$ (8c)

$1 - x_{l+3} \geq 0 \ (l = 2N + 4(k-1) + 2, k = 1, 2, ..., M)$ (8d)

$1 + x_{l+2} + x_{l+3} \geq 0 \ (l = 2N + 4(k-1) + 2, k = 1, 2, ..., M,)$ (8e)

$1 - x_{l+2} + x_{l+3} \geq 0 \ (l = 2N + 4(k-1) + 2, k = 1, 2, ..., N)$ (8f)

Scalar constrained optimization problem is converted into unconstrained multivariable optimization problem using penalty method. Augmented function is defined as

$$A(x) = e(x) + r(P_{term}) \tag{9}$$

Where

$$P_{term} = \sum_{i=1}^{N} \langle 1 + x_{2i+1} \rangle^2 + \sum_{i=1}^{N} \langle 1 - x_{2i+1} \rangle^2 + \sum_{k=1}^{M} \langle 1 - x_{l+3} \rangle^2 \\ + \sum_{k=1}^{M} \langle 1 + x_{l+2} + x_{l+3} \rangle^2 + \sum_{k=1}^{M} \langle 1 - x_{l+2} + x_{l+3} \rangle^2 \tag{10}$$

r is a penalty parameter having large value.

Bracket function for constraint given in (8b) is stated below:-

$$\langle 1 + x_{2i+1} \rangle = \begin{cases} 1 + x_{2i+1} & ; if \left(1 + x_{2i+1}\right) < 0 \\ 0 & ; if \left(1 + x_{2i+1}\right) \geq 0 \end{cases} (11) \tag{11}$$

Bracket function for constraint given by (8e) is stated below:-

$$\langle 1 + x_{l+2} + x_{l+3} \rangle = \begin{cases} 1 + x_{l+2} + x_{l+3} & if \left(1 + x_{l+2} + x_{l+3}\right) < 0 \\ 0 & if \left(1 + x_{l+2} + x_{l+3}\right) \geq 0 \end{cases} \tag{12}$$

Similarly bracket functions for other constraints given by (8c), (8d) and (8f) are undertaken.

III. DIFFERENTIAL EVOLUTION FOR IIR FILTER DESIGN

Differential Evolution (DE) is a population-based stochastic method. It is applied to minimize performance index. Differential evolution combines simple arithmetical operators with the classical operators of the recombination, mutation, and selection to evolve from a randomly generated starting population to a final solution [28]. The different variants of DE are classified using: DE/α/β/δ. α indicates the method for selecting the parent chromosome that will form the base of the mutated vector. β indicates the number of difference

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vectors used to perturb the base chromosome. δ indicates the recombination mechanism used to create the offspring population. The *bin* acronym indicates that the recombination is controlled by a series of independent binomial experiments [33].

A Parameter Setup

The user selects the key parameters that control the DE, i.e. population size (L), boundary constraints of optimization variables (S), mutation factor (f_m), crossover rate (CR), and the stopping criterion of maximum number of iterations (generations) T_{max}. X_{ij}^t is the jth element of S set of filter coefficients giving i^{th} individual of a population. The set of real IIR digital filter co-efficient (*X*) of all generators is represented as the population. For a system with S filter coefficients, the population is represented as a vector of length , S. If there are L members in the population, the complete population is represented as a matrix shown:

B Initialization of an Individual Population

Set generation t = 0. Initialize a population x_{ij}^t (j =1, 2, …, S; i = 1, 2, …, L) individuals with random values generated according to a uniform probability distribution in the S-dimensional problem space. Initialize the entire solution vector population within the given upper and lower limits of the search space.

$$x_{ij}^t = x_j^{min} + rand()(x_j^{max} - x_j^{min})$$

(j =1, 2, …, S; i =1, 2, …, L)   (13)

where *rand()* is uniform random number between 0 and 1

The vector population may violate inequality constraints. This violation is corrected by fixing them either at lower or at upper limit.

C. Opposition-Based Learning:

Evolutionary optimization methods start with some initial random solutions and try to improve them toward some optimal solution(s). The computation time, among others, is related to the distance of these initial guesses from the optimal solution. It can improve the chance of starting with a better solution by simultaneously checking the opposite solution [23]. By doing this, the better one either guess or opposite guess can be chosen as an initial solution. As per the probability theory, 50% of the time, a guess is farther from the solution than its opposite guess [25]. Therefore, starting with the closer of the two guesses as judged by its objective function has the potential to accelerate convergence. The same approach can be applied not only to initial solutions but also continuously to each solution in the current population [25].

$$x_{i+L,j}^t = x_j^{min} + x_j^{max} - x_{ij}^t$$

(j =1, 2, …, S; i =1, 2, …, L)   (14)

where x_j^{min} and x_j^{max} are lower and upper limits of filter coefficients.

D Evaluation of the Individual Population

The goal is to minimize the objective function. The elements of parent/offspring x_{ij}^t may violate constraint. A penalty term is introduced in the objective function to penalize its objective function value. Objective function is changed to the following generalized form:

$$A_i(x_{ij}) = e_i(x_{ij}) + r(P_{term})$$

(j = 1, 2,..., S; i = 1, 2, , L)   (15)

where the penalty factor is given by (11) and (12).

E Mutation Operation (Differential Operation)

Mutation is an operation that adds a vector differential to a population vector of individuals. The mutation operation using the difference between two randomly selected individuals may cause the mutant individual to escape from the search domain So, ten variations mutation strategies [29] defined below are considered for study:

$$Z_{ij1}^t = P_{R_1 j}^t + f_m\left(x_{R_2 j}^t - x_{R_3 j}^t\right)$$ (16)

$$Z_{ij2}^t = x_{Bj}^t + f_m\left(x_{R_1 j}^t - x_{R_2 j}^t\right)$$ (17)

$$Z_{ij3}^t = x_{ij}^t + f_B\left(x_{Bj}^t - x_{ij}^t\right) + f_m\left(x_{R_1 j}^t - x_{R_2 j}^t\right)$$ (18)

$$Z_{ij4}^t = x_{Bj}^t + f_m\left(x_{R_1 j}^t + x_{R_2 j}^t - x_{R_3 j}^t - x_{R_4 j}^t\right)$$ (19)

$$Z_{ij5}^t = x_{R_5 j}^t + f_m\left(x_{R_1 j}^t + x_{R_2 j}^t - x_{R_3 j}^t - x_{R_4 j}^t\right)$$ (20)

$$Z_{ij6}^t = x_{Bj}^t + f_m\left(x_{Bj}^t - x_{ij}^t\right)$$ (21)

$$Z_{ij7}^t = x_{Bj}^t + f_m\left(x_{Bj}^t - x_{ij}^t - x_{R_1 j}^t - x_{R_2 j}^t\right)$$ (22)

$$Z_{ij8}^t = x_{Bj}^t + f_B\left(x_{Bj}^t - P_{xij}^t\right) + f_m\left(x_{R_1 j}^t - x_{R_2 j}^t\right)$$ (23)

$$Z_{ij9}^t = x_{Bj}^t + f_m\left(x_{Bj}^t + x_{ij}^t - x_{R_1 j}^t - x_{R_2 j}^t\right)$$ (24)

$$Z_{ij10}^t = x_{Bj}^t + f_m\left(x_{Bj}^t - x_{Bj}^{t-1}\right)$$ (25)

(j =1, 2, …, S; i =1, 2, …, L)

where t is the time (generation); R_1, R_2 and R_3 are mutually different integers t from the running index, i, randomly selected with uniform distribution. $f_{m(t)} > 0$ is the mutation factor and is taken from the range [0.4, 1] using chaotic sequence. Better solution out of the ten mutation strategies based on minimum augmented objective function is selected.

$$A_i(Z_{ij}^t) = min\left[A_{i1}(Z_{ij1}^t), A_{i2}(Z_{ij2}^t),..., A_{i10}(Z_{ij10}^t)\right]$$ (26)

$Z_i^t = \left[Z_{i1}^t, Z_{i2}^t, ..., Z_{iS}^t\right]^T$ stands for the position of the ith individual of a mutant vector.

F Recombination Operation

Recombination is employed to generate a trial vector by replacing certain parameters of the target vector by the

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corresponding parameters of a randomly generated donor vector.

For each vector, Z_i^{t+1}, an index $R_5(i) \in \{1, 2, ..., S\}$ is randomly chosen using a uniform distribution, and a trial vector, $U_i^{t+1} = \left[U_{i1}^{t+1}, U_{i2}^{t+1}, ..., U_{iS}^{t+1} \right]^T$

$$U_{ij}^{t+1} = \begin{cases} Z_{ij}^t & if \ (R_4(j) \le CR) \ or \ (j = R_5(i)) \\ P_{ij}^t & if \ (R_4(j) > CR) \ or \ (j \ne R_5(i)) \end{cases}$$

$(j = 1, 2, …, S; i = 1, 2, …, L)$ (27)

where

$R_4(j)$ is the jth evaluation of a uniform random number generation with [0, 1]. CR is the crossover or recombination rate in the range [0, 1].

Usually, the performance of a DE algorithm depends on three variables: the population size, the mutation factor $f_m(t)$ and the CR.

G Selection Operation

Selection is the procedure whereby better offspring are produced. To decide whether the vector U_i^{t+1} should be a member of the population comprising the next generation, it is compared with the corresponding vector X_i^t. Thus, if A denotes the objective function under minimization, then

$$x_{ij}^{t+1} = \begin{cases} U_{ij}^{t+1} (j = 1,2,..,S) & ; if \ A(U_i^{t+1}) < A(X_i^t) \\ x_{ij}^t \ \ (j = 1,2,..,S) & ; otherwise \end{cases}$$

$(i = 1, 2, …, L)$ (28)

In this case, the objective A_j of each trial vector U_{ij}^{t+1} is compared with that of its parent target vector x_{ij}^t. If the augmented objective function, A_j of the target vector x_{ij}^t is lower than that of the trial vector, the target is allowed to advance to the next generation. Otherwise, a trial vector replaces the target vector in the next generation.

H. Exploratory Move

In the exploratory move, the current solution denoting filter coefficients is perturbed in positive and negative directions along each variable one at a time and the best point is recorded. The current point is updated to the improved solution at the end of each filter coefficient perturbation. If the point found at the end of all filter coefficient perturbations is different from the original point, the exploratory move is a success; otherwise, the exploratory move is a failure. The filter coefficient x_i is perturbed as follows:

$$x_i^n = x_i^o \pm \Delta_i u_i^j \quad (i = 1,2,...,S; j = 1,2,...,S)$$ (29)

where

$$u_i^j = \begin{cases} 1 & if \ i = j \\ 0 & if \ i \ne j \end{cases}$$ (30)

S denotes number of variables.

The objective function denoted by $A(x_i^n)$ is calculated as follows [35]

$$x_i^n = \begin{cases} x_i^o + \Delta_i u_i & ; A(x_i^o + \Delta_i u_i) < A(x_i^o) \\ x_i^o - \Delta_i u_i & ; A(x_i^o - \Delta_i u_i) < A(x_i^o) \\ x_i^o & ; otherwise \end{cases}$$ (31)

where $(i = 1, 2, ..., S)$ and Δ_i is random for global search and fixed for local search.

The process is repeated till all the filter coefficients are explored and overall minimum is selected as new starting point for next iteration.

J Random Migration Operator

The population diversity and its exploration of the search space are rapidly decreased, and the clustered individuals cannot reproduce newly better individuals by mutation and crossover. In order to increase the exploration of the search space and decrease the selection pressure for a small population, it is randomly selected 0.2L individuals to start migration operation. The jth gene of the ith individual is randomly regenerated as follows [13]:

$$x_{ij}^{t+1} = \begin{cases} x_{bj}^{t+1} + R_i(x_j^{min} - x_{bj}^{t+1}) & if \ \delta < \dfrac{x_{bj}^{t+1} - x_j^{min}}{x_j^{max} - x_j^{min}} \\ x_{bj}^{t+1} + R_i(x_j^{max} - x_{bj}^{t+1}) & if \ otherwise \end{cases}$$ (32)

where x_{bj}^{t+1} is the best individual. R_i and δ are uniform random number.

K Verification of the Stopping Criterion

Generation number is updated, (t = t + 1). Procedure is repeated until a stopping criterion is met, usually a maximum number of iterations (generations), T_{max}. The stopping criterion depends on the type of problem.

L. ALGORITHM: IIR Filter Design Using differential Evolution Algorithm

The search procedure of the proposed differential evolution method has been outlined below.

1 Read data viz. Population size (L), mutation factor (f_m), crossover rate (CR), and the stopping criterion of maximum number of iterations (generations) T_{max}, seed number, S_e, x_j^{min} and x_j^{max} (i = 1, 2, ..,S) etc.

2. Generate an array of (S×L) size of uniform random numbers.
FOR j = 1 to L
    FOR i = 1 to S

3.1 Generate the initial population individual, x_{ij}^0 using (13)

3.2 Compute augmented objective function $A_i(x_{ij}^0)$ using (15).

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3.2 Generate the initial population individual using opposition, $x_{i+L,j}^0$ using (14)

3.3 Compute augmented objective function $A_{i+L}(x_{i+L,j}^0)$ using (14).

END FOR

END FOR

4. Arrange A_j in ascending order and select first L population members out of 2L members.

5. Set iteration counter, t = 0

6. Increment the iteration counter, t = t + 1

7. Select best member A^{best} and corresponding x_{Bj}^t

FOR j = 1 to L

8.1 Generate an array of uniform random numbers and generate five different integer random numbers within the range of 1 to L.

8.2. Apply mutation operation to compute Z_{ijk}^t $(i = 1,2,...,S; k = 1,2,...,10)$ using (16 to 25)

8.3. Compute augmented objective function $A_{ik}(Z_{ijk}^t)$ using (15).

8.4. Find the mutant vector based on minimum augmented objective function using (26) to get $A_i(Z_{ij}^t)$ and corresponding Z_{ij}^t

END FOR

FOR j=1 to L

9.1 Generate arrays R4 and R5 of random numbers of size, 2L

9.2. Apply recombination operation to compute U_{ij}^{t+1} using (27).

9.3. Apply selection operation to compute variable x_{ij}^{t+1} $(i = 1,2,...,S)$ using (28) and augmented objective function $A_i(x_{ij}^{t+1})$ using (15).

9.4. Apply exploratory move to improve the population by implementing algorithm 1.

9.5. Apply random migration to compute variable using (32) and augmented objective function using (15).

END FOR

10. IF $(t < T_{max})$ THEN GOTO 6

11. STOP.

IV. DESIGN OF IIR FILTERS AND COMPARISONS

The design of cascaded digital IIR filter has been implemented by evaluating filter coefficients using differential evolution. Low-pass (LP), high-pass (HP), band-pass (BP) and band-stop (BS) filters have been considered for the design. To design digital IIR filter, 200 equally spaced points are set within the frequency domain $[0,\pi]$,

A. Low-Pass (LP) Filter:

T.he predefined range of pass-band and stop-band are taken as $0 \le \omega \le 0.2\pi$ and $0.3\pi \le \omega \le \pi$, respectively. The value of N and M has been taken as 1. The maximum number of iterations has been taken as 200 for DE. The maximum migration value is 50. The rate of opposition varies between 0 and 1 and has been taken as 0.6. The mutation ratio f_m and crossover ratio CR, has been taken as 0.85 and 0.25, respectively. Exploratory move is repeated 20 times.

The low pass IIR filter model designed by the DE approach is given below in equation (33).

$$H_{LP}(z) = 0.036514 \times \frac{(z+0.89251)(z^2-0.27986\ z+0.94810)}{(z-0.66080)(z^2-1.39399\ z+0.72953)}$$
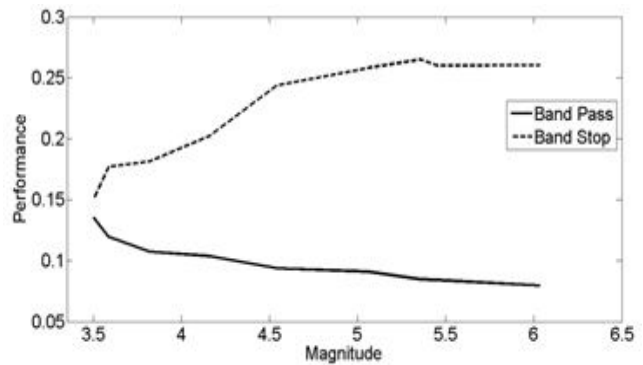
(33)

Figure1: Magnitude with respect to band-pass ripples and band-stop ripples graph of Low-Pass Filter

The variation in magnitude with respect to the ripples of pass-band and stop-band for LP filter has been drawn from 100 iterations in Fig. 1. The minimum value of magnitude is stabilized after 80 generations as shown in Fig. 2. Out of 100 iterations run, the obtained best values for LP filter design are given in Table I. The pass-band and stop-band performance has been obtained from (6) and (7) respectively and are given in Table I.
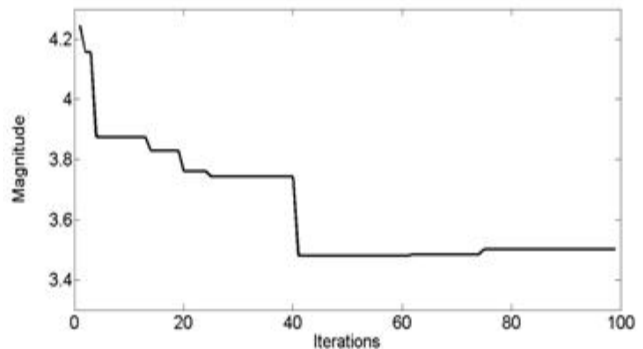

Figure 2: Magnitude with respect to Iteration Graph of LP Filter

The magnitude response of LP filter with respect to normalized frequency has been shown in Fig. 3. The pole-zero graph of low-pass filter has been depicted in Fig. 4 which shows the stability of the designed filter. The magnitude stabilizes after 80 generations.

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TABLE I. DESIGN RESULT FOR LP FILTER

Method	Magnitude Error	Pass-band performance	Stop-band performance
DE	3.5014	$0.88389 \leq |H(e^{j\omega})| \leq 1.019$ (0.1353)	$|H(e^{j\omega})| \leq 0.1505$ (0.1505)
HGA [8]	4.3395	$0.8870 \leq |H(e^{j\omega})| \leq 1.009$ (0.1139)	$|H(e^{j\omega})| \leq 0.1802$ (0.1802)
HTGA [21]	4.2511	$0.90004 \leq |H(e^{j\omega})| \leq 1.000$ (0.0996)	$|H(e^{j\omega})| \leq 0.1247$ (0.1247)
TIA [22]	3.8157	$0.8914 \leq |H(e^{j\omega})| \leq 1.000$ (0.1086)	$|H(e^{j\omega})| \leq 0.1638$ (0.1638)

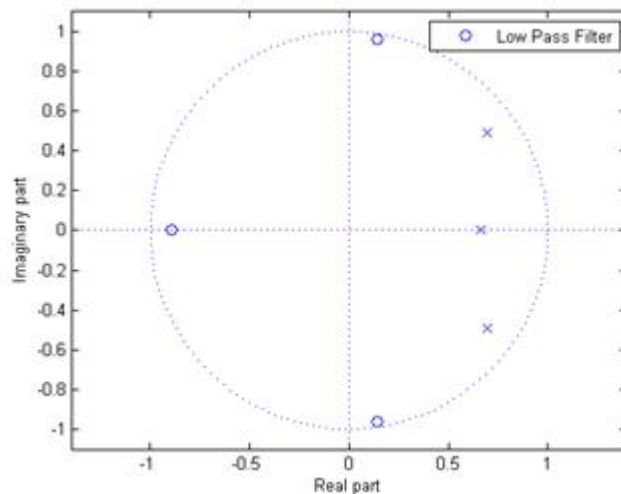Figure 3: Magnitude with respect to normal frequency response of LP filter

Figure-4: Pole-Zero graph of low pass filter

B. High Pass (HP) Filter:

The predefined range of pass-band and stop-band are taken as $0.8\pi \leq \omega \leq \pi$ and $0 \leq \omega \leq 0.7\pi$, respectively. The N and M value has been taken as 1. The maximum number of iterations has been taken as 200 for DE. The maximum migration value is 50. The rate of opposition varies between 0 and 1 and has been taken as 0.6. The mutation ratio f_m and crossover ratio CR, has been taken as 0.85 and 0.25, respectively. Exploratory move is repeated 20 times. The high-

pass IIR filter model designed by the DE approach is given below in equation (34). The obtained best values for HP filter design are given in Table II. The magnitude response of HP filter has been shown in Fig. 5 and the pole-zero graph of high-pass filter has been depicted in Fig. 6 which shows the stability of the designed filter.

$$H_{HP}(z) = 0.015683 \times \left(\frac{z - 0.90131}{z + 0.75157} \right)$$
$$\times \left(\frac{z^2 + 0.56873 \; z + 0.79047}{z^2 + 1.63458 \; z + 0.78028} \right) \quad (34)$$

TABLE II. DESIGN RESULTS FOR HP FILTER

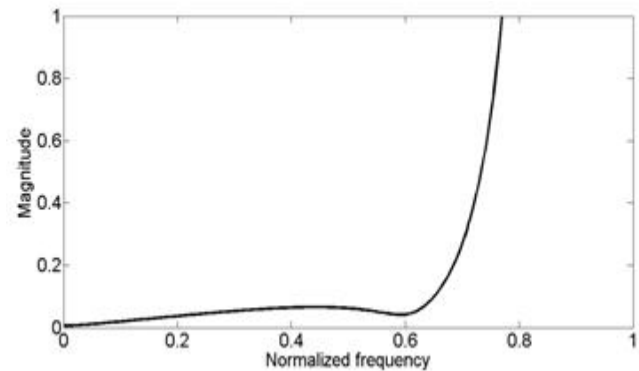| Method	Magnitude Error	Pass-band performance	Stop-band performance
DE	2.8960	$0.89553 \leq |H(e^{j\omega})| \leq 1.014$ (0.11889)	$|H(e^{j\omega})| \leq 0.1100$ (0.1100)
HGA [8]	14.5078	$0.9224 \leq |H(e^{j\omega})| \leq 1.001$ (0.0779)	$|H(e^{j\omega})| \leq 0.1819$ (0.1819)
HTGA [21]	4.8372	$0.9460 \leq |H(e^{j\omega})| \leq 1.000$ (0.0540)	$|H(e^{j\omega})| \leq 0.1457$ (0.1457)
TIA [22]	4.1819	$0.9229 \leq |H(e^{j\omega})| \leq 1.000$ (0.0771)	$|H(e^{j\omega})| \leq 0.1424$ (0.1424)

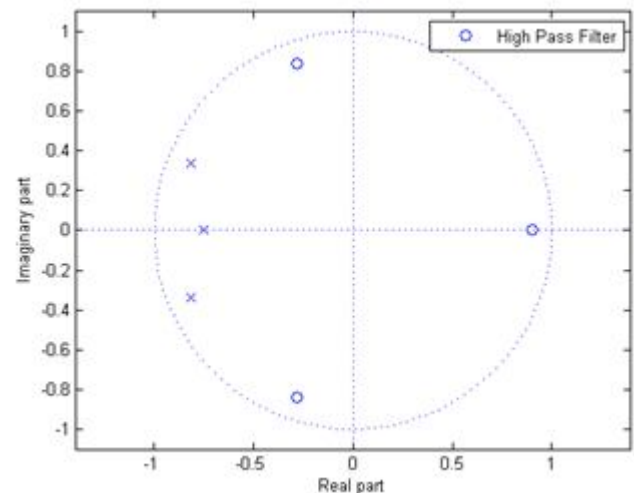Figure 5: Magnitude with respect to normal frequency response of HP filter

Figure 6: Pole-Zero Graph of High Pass Filter

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C. Band Pass (BP) Filter:

The predefined range of pass-band and stop-band are taken as $0.4\pi \leq \omega \leq 0.6\pi$ and $0 \leq \omega \leq 0.25\pi$, $0.75 \leq \omega \leq \pi$, respectively. The value of N and M has been taken as 0 and 3. The maximum number of iterations has been taken as 200 for DE. The maximum migration value is 50. The rate of opposition varies between 0 and 1 and has been taken as 0.6. The mutation ratio f_m and crossover ratio CR, has been taken as 0.85 and 0.25, respectively. Exploratory move is repeated 20 times. The band pass IIR filter model designed by the DE approach is given below in equation (35).

$$H_{BP}(z) = 0.019873 \times \left(\frac{z^2 - 0.00260\ z - 1.36653}{z^2 + 0.00476\ z + 0.55156} \right)$$
$$\times \left(\frac{z^2 - 0.00063\ z - 1.19862}{z^2 - 0.62427\ z + 0.77647} \right)$$
$$\times \left(\frac{z^2 + 0.00074\ z - .922127}{z^2 + 0.62495\ z + 0.77346} \right)$$

$$(35)$$

The obtained best values for BP filter design are given in Table III. The magnitude response of BP filter has been shown in Fig. 7 and the pole-zero graph of band-pass filter has been depicted in Fig. 8 which shows the stability of the designed filter.

TABLE III. DESIGN RESULTS FOR BP FILTER

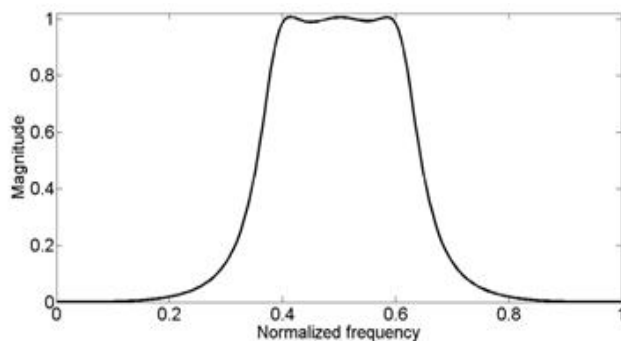| Method	Magnitude Error	Pass-band performance	Stop-band performance
DE	1.2580	$0.9839 \leq \|H(e^{j\omega})\| \leq 1.0065$ (0.0226)	$\|H(e^{j\omega})\| \leq 0.0512$ (0.0512)
HGA [8]	5.2165	$0.8956 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.1044)	$\|H(e^{j\omega})\| \leq 0.1772$ (0.1772)
HTGA [21]	1.9418	$0.9760 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.0234)	$\|H(e^{j\omega})\| \leq 0.0711$ (0.0711)
TIA [22]	1.5204	$0.9681 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.0319)	$\|H(e^{j\omega})\| \leq 0.0679$ (0.0679)

Figure 7: Magnitude with respect to normal frequency response of BP Filter

D. Band Stop (BS) Filter:

The predefined range of pass-band and stop-band are taken as $0 \leq \omega \leq 0.25\pi$, $0.75 \leq \omega \leq \pi$ and $0.4\pi \leq \omega \leq 0.6\pi$, respectively. The N and M values has been taken as 0 and 2. The maximum number of iterations has been taken as 200 for DE. The maximum migration value is 50. The rate of opposition varies between 0 and 1 and has been taken as 0.6. The mutation ratio f_m and crossover ratio CR, has been taken as 0.85 and 0.25, respectively. Exploratory move is repeated 20 times. The band-stop IIR filter model designed by the DE approach is given below in equation.(36) and the best results obtained are given in Table IV.
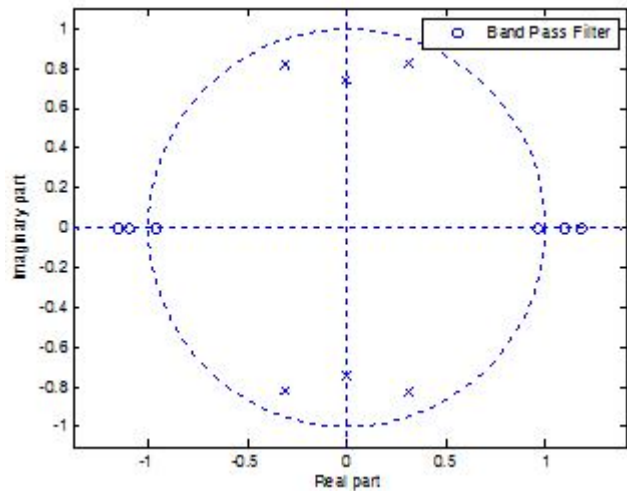

Figure 8: Pole-zero graph of Band-Pass

$$H_{BS}(z) = 0.415583 \times \left(\frac{z^2 + 0.12044\ z + 0.99614}{z^2 + 0.58857\ z + 0.50540} \right)$$
$$\times \left(\frac{z^2 - 0.70514\ z + 0.98496}{z^2 - 0.95940\ z + 0.54561} \right)$$

$$(36)$$

TABLE IV. DESIGN RESULTS FOR BS FILTER

Method	Magnitude Error	Pass-band performance	Stop-band performance
DE	3.1385	$0.92625 \leq \|H(e^{j\omega})\| \leq 1.010$ (0.0838)	$\|H(e^{j\omega})\| \leq 0.1632$ (0.1632)
HGA [8]	6.6072	$0.8920 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.1080)	$\|H(e^{j\omega})\| \leq 0.1726$ (0.1726)
HTGA [21]	4.5504	$0.9563 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.0437)	$\|H(e^{j\omega})\| \leq 0.1013$ (0.1013)
TIA [22]	3.4750	$0.9259 \leq \|H(e^{j\omega})\| \leq 1.000$ (0.0741)	$\|H(e^{j\omega})\| \leq 0.1278$ (0.1278)

The magnitude response of BS filter has been shown in Fig. 9 and the pole-zero graph of band-stop filter has been depicted in Fig. 10 which shows the stability of the designed filter.

The Numerical results are compared with the filter design results given by Tang *et al.* [8], HTGA [21] and TIA [22] for the LP, HP, BP, and BS filters. To design the digital IIR filters, the augmented objective function with the stability constraints, given by (9) is minimized. 1

That is for the digital IIR filters designed under the proposed method gives better performances than the GA-based approach given in HGA [8] , HTGA [21] and TIA [22] approaches. In case of Low-pass filter, it has been observed from Fig.1 that magnitude of ripples in band-stop increases if the magnitude of ripples in band-pass decreases and vice versa.
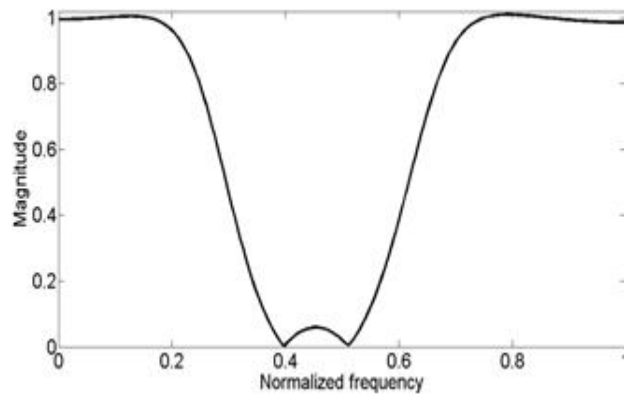

Figure 9: Magnitude with respect to normal frequency response of BS Filter
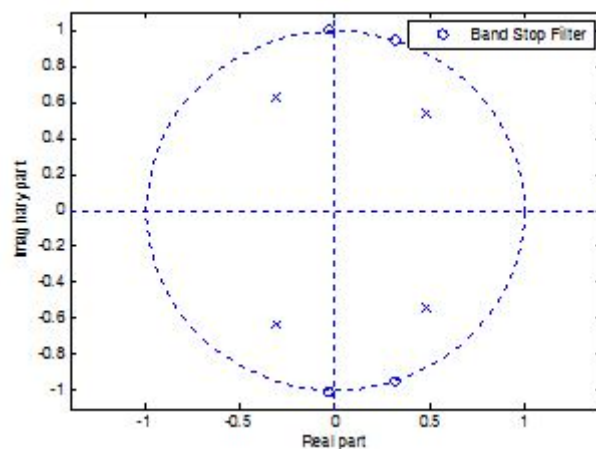

Figure 10: Pole-zero plot of band-stop filter

TABLE V. MAXIMUM, MINIMUM, AVERAGE VALUES AND STANDARD DEVIATION OF MAGNITUDE ERROR

Filter Type	Maximum Magnitude Error	Minimum Magnitude Error	Average Magnitude Error	Standard Deviation of Magnitude Error
Low Pass	6.211293	3.434767	4.54022	0.669059
High Pass	6.234563	2.782686	4.31694	0.997969
Band Stop	5.261713	3.078425	4.039284	0.497461
Band Pass	1.689955	1.2348714	1.42674	0.118483

Similar trend has been observed for HP, BP and BS filters. Moreover, 100 independent trial runs have been given with random seed numbers to have different uniform random numbers for studying the variation in the magnitude.

The maximum value of magnitude error, minimum value of magnitude error, average value of magnitude error and standard deviation in magnitude error are given in Table V The results obtained depict that the standard deviation (SD) is less than 1.0 which shows the robustness of the designed system.

CONCLUSIONS

This paper proposes hybrid differential evolution method for the design of digital IIR filters whereby locally fine tuned by exploratory search method. As shown through simulation results, the DE method works well with an arbitrary random initialization and it satisfies prescribed amplitude specifications consistently. Therefore, the proposed algorithm is a useful tool for the design of IIR filters.

On the basis of above results obtained for the design of digital IIR filter, it can be concluded that the proposed DE method hybridized with exploratory search is superior to the GA-based methods. Further, the proposed DE approach for the design of digital IIR filers allows each filter, whether it is LP, HP, BP, or BS filter, to be independently designed. The proposed DE is very much feasible to design the digital IIR filters, particularly with the complicated constraints. Parameters tuning still is the potential area for further research. The unique combination of exploration search and global search optimization method that is DE provided by the two types of algorithms yields a powerful option for the design of IIR filters.

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